

Bridging Earth and Moon: Balancing Cost, Time, Risk, and Sustainability in Lunar Logistics

Summary

Amid the urgent demands for space exploration and settlement, sustainable and economical space transportation is critical for the development of interstellar civilization. This paper addresses the material transport requirements of a **100,000-person** lunar colony in **2050**, establishes a multi-scenario optimization model to compare the space elevator, rocket and hybrid transport schemes from the dimensions of cost, time, risk and environment, and further proposes targeted optimization strategies.

For Problem 1, a **dual-objective (minimum cost/duration) optimization model** is constructed, which integrates the rocket sub-model (incorporating **TRANSCOST** and latitude correction) and the elevator sub-model (considering maintenance/transport costs). The model is optimized by the **NSGA-II** algorithm, and the results show that: the pure elevator scheme costs **9,991.2B USD** with a duration of **186.22y**; the pure rocket scheme costs **259.6B USD** with a duration of **183.14y** (involving **3,641** annual launches); the hybrid scheme balances the dual objectives, where C1 (the fastest) takes **93.74y** at **9,052.9B USD**, and C2 (the lowest cost) takes **250y** at **2,151.5B USD**.

For Problem 2, a **fault-adjusted optimization model** is built by introducing typical fault factors (elevator cable vibration, thermal fatigue, rocket launch failures). Fault factors are verified to extend the duration and increase costs, with elevators being more sensitive to faults. The adjusted hybrid scheme remains optimal, where the adjusted C1 has a duration of **105.96y** and the adjusted C2 costs **2,565.3B USD**.

For Problem 3, a water cycle model based on the **SIRS** model is constructed, yielding an annual net water demand of **7.51Mt** for the colony (exceeding the capacity of a single system). The cost-optimal hybrid scheme for water transport is determined as: the elevator undertakes **27.5% (2.06Mt)** and rockets undertake **72.5% (5.45Mt)** of the total transport volume, with a total cost of **129.19B USD** (approximately **10** daily rocket launches).

For Problem 4, we adopt a **life cycle assessment (LCA)** framework to monetize six environmental impact indicators. Based on three scenarios: fixed time, fixed cost, and fixed time-cost, the fixed-cost optimized hybrid scheme takes **109.61y**, in which elevators and rockets account for **58.65%** and **41.35%** of the total transport capacity respectively, realizing a balance between environmental benefits and transport efficiency.

Sensitivity and robustness analyses confirm that the proposed models can respond rationally to parameter changes and maintain stability under minor perturbations.

Therefore, the Lunar Colony Administration should prioritize a **hybrid transport strategy**, allocating capacity with a focus on speed during construction and cost-stability during operation.

Keywords:

Lunar colony; Space elevator; Rocket launch; NSGA-II; TRANSCOST; SIRS model; Life Cycle Assessment (LCA)

Contents

1	Introduction	3
1.1	Problem Background	3
1.2	Restatement of the Problem	3
1.3	Our work	3
1.4	Assumptions and Justifications	4
2	Notations	5
3	Dual-Objective (Cost and Time) Optimization Model	6
3.1	Sub-model I: Cost-Time Model of the Space Elevator System	6
3.2	Submodel II: Cost-Time Model for Traditional Rocket Launches	6
3.3	Submodel III: Integrated Transportation System Optimization Model	9
4	Risk Assessment Model for Transportation System Failures	10
4.1	Data Processing	10
4.2	Model Construction: Transportation Model Adjusted for System Failures	11
4.3	Results and Analysis	13
5	Water Resource Supply Model for Lunar Colonization	15
5.1	Data Processing	15
5.2	Model Construction: Water Cycle Dynamics and Resupply Optimization	16
5.3	Water Resource Supply Results and Analysis	17
6	Environmental Impact Minimization Model	18
6.1	LCA-Based Environmental Impact Quantification Model	18
6.2	Environmental Cost Minimization Optimization Model	20
6.3	Environmental Impact Minimization Results and Analysis	21
7	Sensitivity Analysis	22
7.1	Sensitivity Analysis on Rocket Launch Failure Probability	22
7.2	Sensitivity Analysis on Water Recovery Efficiency Rates	23
8	Model Evaluation	23
8.1	Strengths	23
8.2	Weaknesses and Future Research Directions	23
A		25

1 Introduction

1.1 Problem Background

With the growing urgency of humanity's aspirations for space exploration and settlement, coupled with the mounting pressures exerted on Earth's ecosystems, the development of a sustainable and cost-effective space transportation system has emerged as a critical enabler for advancing interstellar civilization, thereby driving research into novel space transportation infrastructure. In recent years, scientists have proposed a disruptive technological concept—the space elevator system—intended to facilitate low-cost, high-efficiency, and environmentally friendly transportation of cargo and personnel, while supporting the establishment of lunar colonies.

Nevertheless, this system remains in the phases of conceptual validation and technological breakthroughs, and its construction and operation may confront multiple risks such as dynamic instability of the tether and structural damage. Concurrently, conventional rocket launch technology continues to evolve, with new-generation heavy-lift launch vehicles demonstrating substantial improvements in orbital payload delivery capacity.

Consequently, when formulating cargo transportation schemes for lunar colonies, it is imperative to comprehensively consider the respective advantages and limitations of space elevator and rocket systems, so as to design an optimal strategy for the transportation of personnel and supplies.

1.2 Restatement of the Problem

Considering the background information and restricted conditions identified in the problem statement, we need to solve the following problems:

- **Problem 1:** Construct a model to evaluate the three material transportation schemes.
- **Problem 2:** Optimize the transportation model under system failures and propose adaptive strategies.
- **Problem 3:** Model the annual water demand and supply cost for 100,000 residents, with timeline planning.
- **Problem 4:** Build a model to analyze the environmental impact on Earth under different transportation scenarios, and propose model optimization schemes to reduce environmental costs.
- **Problem 5:** Write a recommendation letter to MCM on establishing and sustaining a lunar colony.

1.3 Our work

In order to clearly illustrate our work, we draw the flowchart Figure .

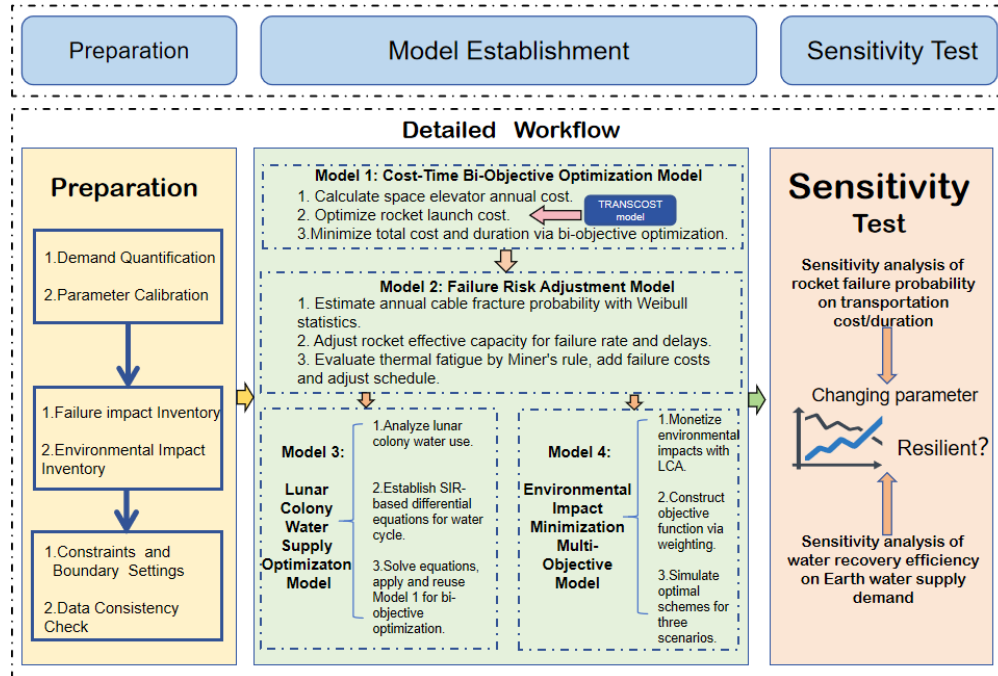


Figure 1: Our work

1.4 Assumptions and Justifications

Considering those practical problems always contain many complex factors, first of all, we need to make reasonable assumptions to simplify the model, and each hypothesis is closely followed by its corresponding explanation:

- **Assumption:** Through 2050, human basic science, materials science and aerospace engineering will advance steadily, with all critical technological breakthroughs achieved as scheduled.

Justification: All model parameters (elevator strength, rocket cost, ISRU efficiency) are predicated on 2050 technological levels, forming the study's logical basis. Unforeseen technological bottlenecks would invalidate the space elevator and advanced rocket-based transportation system proposed herein; technological development is assumed to follow an optimistic yet rational trajectory.

- **Assumption:** Inflation, time value of money (discount rate) and Earth's macroeconomic fluctuations are excluded from cost estimation for the multi-century model time horizon.

Justification: This paper focuses on relative cost and efficiency comparisons of transportation schemes rather than absolute dollar forecasts. Financial factors are omitted to avoid model overcomplexity and overshadowing technical analysis, with relative costs measured in current US dollars.

- **Assumption:** The 100,000-population lunar colony operates as a stable closed system with fixed water demand structure, constant recycling efficiency and no systemic supply disruptions during the planning period.

Justification: The water supply-demand and transportation optimization models rely on steady-state demand. This assumption simplifies social uncertainty, with the colony assumed to have sound governance ensuring stable water cycle operation, focusing the study on technical and logistics optimization.

- **Assumption:** (For Models 1, 2, 3) Transportation system environmental externalities are not rigid project constraints, and only minimized as the objective function in Model 4 (environmental cost minimization model).

Justification: Core models for economic cost and efficiency only consider engineering system direct inputs and outputs. Environmental factors are studied independently in Model 4 to avoid premature elimination of technical pathways, with the first three models assuming an idealized Earth with unrestricted environmental carrying capacity.

- **Assumption:** (For Models 1, 2, 3) The Earth-Moon space environment remains at its long-term average during the planning period, with no extreme space weather causing long-term transportation system paralysis.

Justification: The risk assessment model only considers internal technical failures; extreme external space environment fluctuations are ignored as low-probability events. Such events are unmodelable and catastrophically impact all schemes equally, with no effect on relative comparisons. The transportation system is assumed to withstand routine space environment challenges.

2 Notations

The key mathematical notations used in this paper are listed in Table 1.

Table 1: Notations

Symbol	Description
M_{total}	Total mass of materials to be transported (tons)
γ	Proportion of transportation demand allocated to space elevator
t	Total project duration (years)
$C_{\text{elevator}}(t)$	Total cost of space elevator system over t years (USD)
$C_{\text{rocket}}(\beta, x_i, y_i, t)$	Total cost of rocket system (USD)
$P_{\text{fail, rocket}}$	Single-launch failure probability for rockets
$P_{\text{annual}}^{\text{fracture}}$	Annual fracture probability of space elevator tether
$P_{\text{annual}}^{\text{thermal}}$	Annual thermal fatigue failure probability of elevator
η_h, η_a	Domestic and agricultural wastewater recovery efficiency
$\text{Input}_{\text{Earth}}(t)$	Water resupply needed from Earth over time t (tons)

* Some variables are not listed. Their specific meanings will be introduced below.

3 Dual-Objective (Cost and Time) Optimization Model

To systematically evaluate and compare the performance of different transportation schemes, this paper develops a dual-objective optimization model for lunar logistics, with the objectives of minimizing total cost and total transportation time. The framework consists of three components: cost-time analysis of the space elevator system, cost-time assessment of conventional rocket launches, and a dual-objective optimization model integrating both transportation modes.

3.1 Sub-model I: Cost-Time Model of the Space Elevator System

The annual total cost of the space elevator system comprises fixed maintenance costs and variable transportation costs:

- **Fixed maintenance cost:** Covers annual operating expenses such as cable monitoring, equipment maintenance, and site management, set at $C_{\text{main}} = 10^9$ USD/year.
- **Variable transportation cost:** Proportional to the annual transported mass, including energy, wear, and labor expenses, with a unit cost of $C_{\text{var}} = 2.2 \times 10^5$ USD/ton .

The annual total cost is expressed as:

$$C_{\text{annual}} = C_{\text{main}} + C_{\text{var}} \times M_{\text{annual}} \quad (1)$$

where M_{annual} denotes the annual transported mass (tons/year).

For a total demand M_{total} (tons) transported uniformly over t years:

$$M_{\text{annual}} = \frac{M_{\text{total}}}{t} \quad (2)$$

Substituting Equation (2) into Equation (1) yields the total cost as a function of t :

$$C_{\text{elevator}}(t) = \left(C_{\text{main}} + \frac{M_{\text{total}} \times C_{\text{var}}}{t} \right) \times t \quad (3)$$

Given the maximum annual transportation capacity $\text{Cap}_{\text{SE}} = 5.37 \times 10^5$ tons/year and our objective of minimizing project duration, we have:

$$t = \frac{M_{\text{total}}}{5.37 \times 10^5} \quad (4)$$

3.2 Submodel II: Cost-Time Model for Traditional Rocket Launches

The total cost evaluation of traditional rocket launches aims to minimize the project duration and total capital expenditure. We first establish a cost model for a single launch at a single launch site, and then construct a system-level optimization model integrating the decision-making of launch site selection, launch frequency and project duration.

3.2.1 Single Launch Cost Model with Latitude Correction

Based on the cost-benefit model of reusable rockets constructed by Kang et al. within the TRANSCOST framework, this study introduces a latitude correction factor to expand its applicability, and makes the assumption that rocket manufacturing technology will reach a mature stage by 2050, thus no additional R&D amortization costs are considered in the model.

Under this circumstance, the baseline single launch cost C_{base} of reusable rockets (excluding the latitude effect) can be decomposed into four major cost modules: manufacturing, reuse, operation and insurance, with the expression as follows:

$$C_{\text{base}} = C_{m,\text{avg}} + C_{re,\text{per}} + C_{op,\text{per}} + C_{ins,\text{per}}$$

The calculation method for each module is given below:

1. Average manufacturing cost $C_{m,\text{avg}}$

$$C_{m,\text{avg}} = \frac{C_{fu} \sum_{n=1}^{N_m} n^p}{N_{\text{total}}}, \quad p = \frac{\log f_{lc}}{\log 2}$$

where the first-unit manufacturing cost $C_{fu} = MY_{m,r} \cdot C_{MY}$, and the engineering man-years required for manufacturing $MY_{m,r}$ is defined as:

$$MY_{m,r} = 1.8 \times 0.45 M_p^{0.66} \cdot f_1 f_2 f_3 f_8 \cdot \left(\frac{N_{re}}{10} \right)^{0.3}$$

M_p denotes the payload mass, N_{re} is the maximum reuse times of a single rocket, and f_1, f_2, f_3, f_8 are the technical and experience correction factors; C_{MY} represents the unit man-year cost, f_{lc} is the manufacturing learning curve factor, N_m is the total number of manufactured rocket bodies, and N_{total} is the total number of launches in the project.

2. Per-launch reuse cost $C_{re,\text{per}}$ and per-launch operation cost $C_{op,\text{per}}$

- **Reuse cost:** It covers the refurbishment, inspection and testing of rocket bodies after recovery:

$$C_{re,\text{per}} = 0.03 C_{fu} \cdot k^q, \quad q = \frac{\log f_{lc,\text{reuse}}}{\log 2}$$

where k is the number of completed reuse times of the rocket body, and $f_{lc,\text{reuse}} \in [1.1, 1.2]$ is the reuse learning curve factor.

- **Operation cost:** It includes transportation, propellant, launch measurement and control, etc.:

$$C_{op,\text{per}} = 1.5 \times C_{op,b} \cdot \left(\frac{M_p}{M_{p,b}} \right)^{0.75} \cdot N_l^s, \quad s = \frac{\log f_{lc,\text{op}}}{\log 2}$$

where $C_{op,b}$ is the baseline operation cost corresponding to the reference payload $M_{p,b}$, N_l is the cumulative number of launches at the launch site, and $f_{lc,\text{op}} \in [0.9, 0.95]$ is the operation learning curve factor.

3. Per-launch insurance cost $C_{ins,\text{per}}$: Accrued at 10% of the average manufacturing cost.

$$C_{ins,\text{per}} = 0.1 C_{m,\text{avg}}$$

On the basis of the baseline cost, we introduce a latitude correction factor $f(\varphi)$ to quantify the impact of the launch site latitude φ on launch efficiency, including the velocity gain brought by the Earth's rotation and the requirement for orbital inclination adjustment:

$$f(\varphi) = 1 + k_1(1 - \cos \varphi) + k_2 \sin^2 \frac{\varphi}{2}$$

where $k_1 \in [0.25, 0.35]$ and $k_2 \in [0.10, 0.15]$ are the latitude sensitivity coefficients. Accordingly, the **single launch cost** at the i -th launch site (with latitude φ_i) is given by:

$$C_{\text{single},i} = C_{\text{base}} \cdot f(\varphi_i)$$

3.2.2 Multi-Objective Optimization Total Cost Model

The multi-objective optimization model based on the single launch cost model is given with decision variables:

- x_i : Launch site selection (1 if the i -th site is selected, 0 otherwise)
- y_i : Annual launch count at the i -th site
- t : Project cycle (years)
- β : Proportion of total cargo demand handled by rockets
- $C_{f,l,i}$: Annual fixed cost of the i -th launch site

Objective Functions

The annual total cost combines fixed and variable costs:

$$C_{\text{annual}} = \sum_{i=1}^n x_i C_{f,l,i} + \sum_{i=1}^n y_i C_{\text{single},i}$$

The project total cost is:

$$C_{\text{total}} = C_{\text{annual}} \cdot t$$

The optimization objectives are to minimize the project total cost $C_{\text{rocket}}(\beta, x_i, y_i, t) = C_{\text{total}}$ and the project cycle t .

Constraints

1. **Cargo demand constraint:** The total payload delivered over the project cycle must meet the rocket-assigned demand:

$$\sum_{i=1}^n y_i M_p t \geq \beta M_{\text{total}}$$

2. **Launch site constraint:** Annual launches at each site cannot exceed its maximum capacity, and only selected sites can launch:

$$0 \leq y_i \leq Y_{\text{max}} x_i, \quad x_i \in \{0, 1\}, \quad \forall i$$

3. **Project cycle lower bound:** The project cycle must not be shorter than the ratio of total demand to the system's maximum annual launch capacity:

$$t \geq \frac{\beta M_{\text{total}}}{\sum_{i=1}^n x_i Y_{\text{max}} M_p}$$

3.3 Submodel III: Integrated Transportation System Optimization Model

This scheme develops a collaborative transportation model for the space elevator and rocket launch systems, targeting the simultaneous minimization of **total cost** and **total project duration**. An allocation proportion variable $\gamma \in [0, 1]$ (the proportion of transportation demand borne by the space elevator system) is introduced to characterize the differentiated transportation volume allocation between the two systems. Thus, the total transportation demand M_{total} in Submodel I and Submodel II is adjusted to γM_{total} and $(1 - \gamma)M_{\text{total}}$, respectively. The total project duration is the maximum of the completion times of the two subsystems:

$$t = \max(t_{\text{elevator}}, t_{\text{rocket}})$$

A bi-objective coupled optimization model for minimizing total cost and total project duration is formulated as:

$$\begin{cases} \min_{\gamma, x_i, y_i, t} & C_{\text{total}} = C_{\text{elevator}}(\gamma, t) + C_{\text{rocket}}(1 - \gamma, x_i, y_i, t) \\ \min_{\gamma, x_i, y_i, t} & t \\ \text{s.t.} & 0 \leq \gamma \leq 1; \quad \text{All constraints of Submodel I and Submodel II} \end{cases}$$

where $C_{\text{elevator}}(\gamma, t)$ is calculated using Equation (1) in Submodel I, and $C_{\text{rocket}}(1 - \gamma, x_i, y_i, t)$ is derived from the rocket optimization model in Submodel II.

Based on the bi-objective optimization model, numerical solutions for the three transportation strategies are obtained using the NSGA-II algorithm, with key results summarized in Table 2.

Table 2: Key Numerical Results of Transportation Schemes

Indicator	Elevator-Only	Rocket-Only	Hybrid (Min Time)	Hybrid (Min Cost)
Total cost (B\$)	9991.21	2596.02	9052.86	2151.53
Total duration (years)	186.22	183.14	93.74	250.00
Payload allocation (E/R)	100% / 0%	0% / 100%	49.3% / 50.7%	1.0% / 99.0%

The optimization results in Table 2 are analyzed as follows:

- **Cost performance analysis:** The minimum-cost hybrid scheme (C2) achieves the lowest net present value, representing a 78.5% cost reduction compared with the elevator-only scheme and a 17.1% reduction compared with the rocket-only scheme.
- **Time performance analysis:** The minimum-time hybrid scheme (C1) reduces the total project duration by approximately 50% relative to both single-mode schemes.

Optimization results for schemes involving rocket launches show that all ten available launch sites are utilized. Although low-latitude sites offer theoretical velocity advantages from Earth's rotation, annual launch frequencies are distributed relatively uniformly across sites (e.g., 361365 launches per site per year in Scheme B). This uniformity arises because the need for full system

capacity utilization outweighs the marginal cost advantages of latitude differences (less than 5% of total launch costs) under the high annual launch demand of thousands of missions.

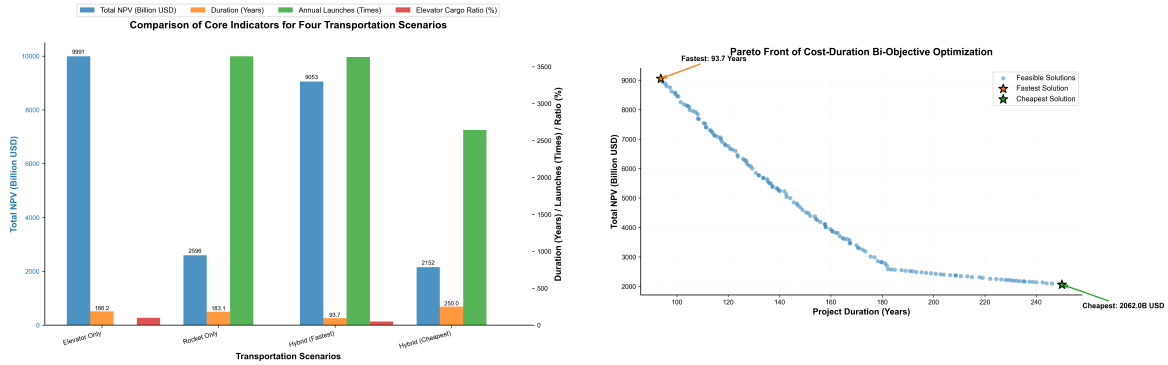


Figure 2: Problem 1 Results

Based on the optimization results and analysis, we conclude:

- The elevator-only scheme is economically inefficient and slow, while the rocket-only scheme, though moderately cost-effective, demands unsustainably high launch rates that threaten reliability.
- Hybrid schemes balance cost and time effectively: Scheme C1 minimizes project duration, and Scheme C2 minimizes total cost.

4 Risk Assessment Model for Transportation System Failures

4.1 Data Processing

Before addressing the overall problem, we quantify the failure probabilities of critical components in the three categories of risks. By referencing historical failure rates of Falcon-series rockets over the past decade, we find that the single-launch failure probability of heavy-lift rockets $P_{\text{fail, rocket}}$ is extremely low, not exceeding 1%. We therefore assume that, due to technological advancements, $P_{\text{fail, rocket}}$ will remain at this low level (approximately 0.18%) beyond 2050. Thus, we focus primarily on the failure probabilities of the following two critical components and construct two corresponding assessment models: the first is a model for calculating the fracture probability of the space elevator tether under large-amplitude oscillations, and the second is a model for the thermal fatigue failure probability of the elevator system based on thermal cycling.

These probability values are directly substituted into the subsequent cost model to calculate the additional maintenance costs and transportation capacity losses caused by system failures.

4.1.1 Submodel 1: Tether Fracture Probability Model Based on Dynamic Vibration

This model calculates the annual tether fracture probability under resonance using the Weibull distribution fitted to graphene strength experimental data. The maximum nominal static stress of the tether at the geostationary orbit is

$$\sigma_{\text{static}}^{\text{max}} = 63 \text{ GPa},$$

and lateral vibrations induced by lunar and solar gravitational perturbations amplify the dynamic stress to

$$\sigma_{\text{dynamic}} = k \cdot \sigma_{\text{static}}^{\max},$$

where $k \in [1.000, 1.1]$ denotes the resonant stress amplification factor. Considering the strength dispersion caused by nanoscale defects, the single-event fracture probability under dynamic stress loading is

$$P_{\text{single}} = 1 - \exp \left[- \left(\frac{\sigma_{\text{dynamic}}}{\sigma_f} \right)^m \right],$$

in which $\sigma_f \in [30, 100]$ GPa is the characteristic tensile strength and $m = 2.7$ is the Weibull modulus. For $N \in \{1, 12, 52\}$ independent resonant events per year, the annual fracture probability is given by

$$P_{\text{annual}}^{\text{fracture}}(\sigma_f, k, N) = 1 - \exp \left[-N \left(\frac{k \cdot 63}{\sigma_f} \right)^{2.7} \right]. \quad (5)$$

Assuming each fracture results in a complete system shutdown, the annual efficiency loss rate is expressed as

$$\eta_{\text{sway}} = \frac{P_{\text{annual}}^{\text{fracture}} \times T_{\text{repair}}}{365}. \quad (6)$$

4.1.2 Submodel 2: Thermal Fatigue Failure Probability Model for the Space Elevator System

This model estimates the system's annual thermal fatigue failure probability based on Miner's linear cumulative damage rule and experimental data. The baseline material has a thermal fatigue failure cycle of 1483 in the space thermal environment (Anvari, 2018), and graphene-reinforced composites with 10% improved fatigue performance yield

$$N_{\text{failure, Graphene}} = 1483 \times 1.1 = 1631.3.$$

The annual thermal cycles of the tether, induced by Earth shadow transits, solar irradiation variations and seasonal temperature fluctuations, are engineering-estimated as

$$n_{\text{annual}} = 13.36 \text{ cycles/year}.$$

Per Miner's rule, annual cumulative damage is taken as the conservative upper bound of annual failure probability, giving

$$P_{\text{annual}}^{\text{thermal}} = \frac{n_{\text{annual}}}{N_{\text{failure}}}. \quad (7)$$

The annual equivalent failure probability of graphene-reinforced composites is thus calculated as

$$P_{\text{annual}}^{\text{thermal}} = \frac{13.36}{1631.3} \approx 0.00819 \quad (0.819\%).$$

4.2 Model Construction: Transportation Model Adjusted for System Failures

The ideal-condition transportation model in Problem 1 is revised to account for practical failure risks (tether sway, mechanical and launch failures), which directly impact effective transportation capacity, project duration and total cost.

4.2.1 Submodel 1: Space Elevator System Adjusted for Failures

The space elevator has two main failure modes: tether dynamic sway and fracture/thermal fatigue-induced system failure, which reduce operational efficiency, incur extra maintenance costs, extend project duration and raise total costs.

The effective annual transportation capacity of the space elevator, considering sway-induced efficiency loss and failure maintenance downtime, is:

$$\text{Cap}_{\text{SE, eff}} = \text{Cap}_{\text{SE}} \cdot (1 - \eta_{\text{sway}}) \cdot \text{ASE} \cdot (1 - P_{\text{fail, rocket}})$$

where $\text{ASE} = 1 - P_{\text{annual}}^{\text{thermal}} \cdot T_{\text{repair}}$ is the system availability coefficient, and T_{repair} denotes the mean time to repair for a single failure.

The adjusted project duration for delivering the elevator-allocated mass γM_{total} is:

$$t_{\text{elevator}} = \frac{\gamma M_{\text{total}}}{\text{Cap}_{\text{SE, eff}}}$$

The adjusted total cost of the space elevator system comprises the baseline cost (from Problem 1) and additional risk costs:

$$C_{\text{elevator, adj}}(\gamma, t) = C_{\text{elevator, base}}(\gamma, t) + C_{\text{repair, elevator}} + C_{\text{loss, elevator}} \quad (8)$$

where the repair cost is $C_{\text{repair, elevator}} = P_{\text{fail, SE}} \cdot C_{\text{repair}}$ (C_{repair} is the average repair cost per failure), and $C_{\text{loss, elevator}}$ (cargo loss cost) is defined in the next submodel.

4.2.2 Submodel 2: Conventional Rocket Launch System Adjusted for Failures

For each launch site i , the effective annual delivered mass, accounting for launch failures and operational delays, is:

$$\text{Rate}_{\text{eff}}^{(i)} = y_i \cdot M_p \cdot (1 - P_{\text{fail, rocket}}) \cdot \left(1 - \frac{y_i \cdot P_{\text{fail, rocket}} \cdot T_{\text{delay}}}{365}\right)$$

where $P_{\text{fail, rocket}}$ is the single-launch failure probability (from Falcon-series historical data) and T_{delay} is the average post-failure delay time.

The total effective annual delivered mass of the adjusted rocket system is:

$$\text{Cap}_{\text{rocket, eff}} = \sum_{i=1}^n \text{Rate}_{\text{eff}}^{(i)}$$

The adjusted project duration for delivering the rocket-allocated mass $(1 - \gamma)M_{\text{total}}$ is:

$$t_{\text{rocket}} = \frac{(1 - \gamma)M_{\text{total}}}{\text{Cap}_{\text{rocket, eff}}}$$

The adjusted total rocket system cost includes the baseline cost (from Problem 1) and launch failure-induced cargo loss cost:

$$C_{\text{rocket, adj}}(1 - \gamma, x_i, y_i, t) = C_{\text{rocket, base}}(1 - \gamma, x_i, y_i, t) + C_{\text{loss, rocket}} \quad (9)$$

where the cargo loss cost is $C_{\text{loss, rocket}} = \frac{(1 - \gamma)M_{\text{total}} \cdot P_{\text{fail, rocket}}}{1 - P_{\text{fail, rocket}}} \cdot C_{\text{cargo}}$, and $C_{\text{cargo}} = C_{\text{single}}/M_p$ is the unit cargo transportation cost (per ton).

4.2.3 Integrated Bi-Objective Optimization Model Adjusted for Failures

The objective functions, optimization variables and constraints remain consistent with Problem 1, with only the total costs of the space elevator and rocket systems replaced by the above failure-adjusted costs (including all risk-induced additional costs).

The revised bi-objective optimization model is:

$$\begin{cases} \min_{\gamma, x_i, y_i, t} & C_{\text{total, adj}} = C_{\text{elevator, adj}}(\gamma, t) + C_{\text{rocket, adj}}(1 - \gamma, x_i, y_i, t) \\ \min_{\gamma, x_i, y_i, t} & t_{\text{total}} = \max(t_{\text{elevator}}, t_{\text{rocket}}) \end{cases} \quad (10)$$

4.3 Results and Analysis

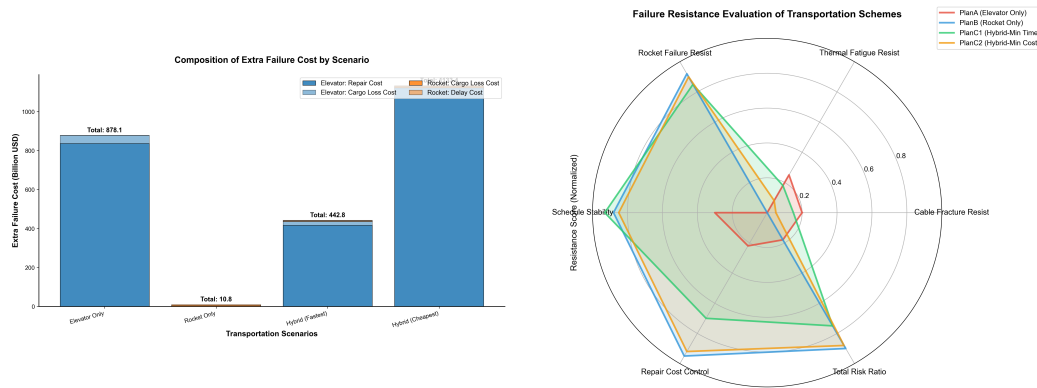


Figure 3: Problem 1 Results

Based on numerical solutions of the failure risk-adjusted model, we evaluate the performance of four transportation schemes (pure space elevator, pure conventional rocket, two hybrid strategies) and analyze the impacts of system failures on project duration and total cost. Key results are summarized in Table 3.

Table 3: Key Performance Indicators of Transportation Schemes with Failure Adjustment

Performance Indicator		Scheme A (Pure Elevator)	Scheme B (Pure Rocket)	Scheme C1 (Hybrid, Min Time)	Scheme C2 (Hybrid, Min Cost)
Total Cost (billion USD)		10388.24	2585.06	9407.72	2565.29
Total Duration (years)		213.53	190.54	105.96	255.67
Payload Allocation (Elev./Rocket)		100% / 0%	0% / 100%	49.3% / 50.7%	1.0% / 99.0%
Duration Delay (years)		27.31	5.72	16.43	6.01
Additional Failure Cost (billion USD)		398.10	4.81	288.48	416.05

4.3.1 Impact of Failures on Project Duration

System failures extend the transportation timeline of all schemes, with the space elevator system showing significant sensitivity. Key results are as follows:

- Scheme A (Pure Elevator): Longest delay of 27.31 years (14.67%), driven by tether sway and thermal fatigue.
- Scheme B (Pure Rocket): Moderate delay of 5.72 years (3.09%), caused by launch failures and subsequent delays.
- Scheme C1 (Hybrid, Min Time): Maintains the shortest overall duration (105.96 years) despite a total delay of 16.43 years.
- Scheme C2 (Hybrid, Min Cost): Limited delay of 6.01 years with negligible impact from the elevator subsystem.

4.3.2 Impact of Failures on Total Cost

Failure-induced additional costs are prominent in elevator-dependent schemes, with core findings listed below:

- Scheme A (Pure Elevator): Highest additional failure cost of 398.10 billion USD (3.83% of total cost) from maintenance and capacity losses.
- Scheme B (Pure Rocket): Minimal additional cost of 4.81 billion USD (0.19% of total cost), mainly due to payload losses.
- Scheme C1 (Hybrid, Min Time) & C2 (Hybrid, Min Cost): Additional failure costs reach 288.48 and 416.05 billion USD respectively, with the elevator subsystem as the primary cost driver.

4.3.3 Launch Site Configuration Analysis

- All ten available launch sites are utilized for rocket-involved schemes with uniform launch frequency distribution, reflecting a global resource utilization balance under high-intensity launch demand.

4.3.4 Conclusions

Comprehensive assessment leads to the following key conclusions:

- The pure space elevator scheme is uncompetitive in economy and timeliness, and highly failure-sensitive, making it unsuitable as the primary transportation mode.
- The pure rocket scheme features good cost-effectiveness and failure robustness but requires an extremely high annual launch frequency (>3600 missions), challenging system reliability and sustainability.
- Hybrid transportation strategies achieve an effective balance between cost and duration: Scheme C1 (106 years) is optimal for minimum project duration, while Scheme C2 (256.53 billion USD) is optimal for minimum total cost.

5 Water Resource Supply Model for Lunar Colonization

5.1 Data Processing

Water consumption in a lunar colony is primarily divided into three categories: domestic water, agricultural water, and technical water. Technical water refers to water used for equipment cooling, firefighting, and other purposes, accounting for only 2%. In addition to direct consumption, water resources can be acquired on the Moon to supplement the consumed water, relying mainly on two approaches: Earth resupply and In-Situ Resource Utilization (ISRU).

5.1.1 Basic Water Demand of the Colony

Based on the research results of Jeffrey S. Lee et al., the annual water demand data for a lunar colony with a scale of 100,000 people can be obtained, as shown in the following table.

Table 4: Annual Water Demand of a Lunar Colony with a Scale of 100,000 People

Demand Type	Annual Water Consumption for 100,000 People (tons/year)	Proportion
Domestic Water	1.52×10^6	18%
Agricultural Water	8.90×10^6	80%
Technical Water	2.48×10^5	2%
Total Annual Demand	1.07×10^7	100%

5.1.2 Major Water Supply Sources for the Lunar Colony

The water supply for a lunar colony primarily originates from three sources: water recovered from the wastewater recycling system, water produced by In-Situ Resource Utilization (ISRU) technology, and water transported from Earth.

The first source is water recovered from the wastewater recycling system. The domestic wastewater recovery efficiency is $\eta_h = 0.95$, and the agricultural wastewater recovery efficiency is $\eta_a = 0.65$.

The second source is water obtained via In-Situ Resource Utilization (ISRU) technology. ISRU is the core technology for acquiring water resources by mining water ice at the lunar poles. This study adopts the 3D numerical thermal drilling model proposed by Brisset et al., with its key parameters as follows:

Production Capacity of a Single Thermal Drilling Device:

$$R_{\text{drill}} = 5.256 \text{ tons/year}$$

Required Number of Thermal Drilling Devices: In this analysis, the number of drills is assumed to be $N = 1000$ units. The impact of this parameter will be examined in a subsequent sensitivity analysis.

Annual Water Production via ISRU:

$$\text{Water}_{\text{ISRU}} = N \cdot R_{\text{drill}}$$

The third source, which is also the focus of this analysis, is water transported from Earth using the transportation system. The total volume of water resources that must still be transported from Earth to the Moon, in addition to the water obtained via ISRU technology and the wastewater recycling system, is denoted by $\text{Input}_{\text{Earth}}(t)$.

5.2 Model Construction: Water Cycle Dynamics and Resupply Optimization

To characterize the dynamic process of the water resource cycle in the lunar colony, we draw upon the framework of the Susceptible-Infected-Recovered (SIR) model from infectious disease dynamics and establish the following system of differential equations for the water cycle:

$$\begin{cases} \frac{dS}{dt} = Input(t) + k_{out}(\eta_h R_h + \eta_a R_a) - (\beta_h + \beta_a + \beta_t) \\ \frac{dI_h}{dt} = \beta_h - k_{in} I_h; \frac{dI_a}{dt} = \beta_a - k_{in} I_a \\ \frac{dR_h}{dt} = \eta_h k_{in} I_h - k_{out} R_h; \frac{dR_a}{dt} = \eta_a k_{in} I_a - k_{out} R_a \end{cases} \quad (11)$$

The parameters in the above system of differential equations are explained as follows:

- $S(t)$: Volume of available clean water resources;
- $I_h(t), I_a(t)$: Volumes of untreated domestic and agricultural wastewater, respectively;
- $R_h(t), R_a(t)$: Volumes of treated recycled domestic and agricultural water resources, respectively;
- $Input(t)$: Volume of externally resupplied water resources;
- $\beta_h, \beta_a, \beta_t$: Consumption rates of domestic, agricultural, and technical water, respectively;
- Recovery efficiency: $\eta_h = 0.95, \eta_a = 0.65$;
- Wastewater transfer coefficient: $k_{in} = 0.0274 \text{ year}^{-1}$ (domestic wastewater), 0.0055 year^{-1} (agricultural wastewater);
- Treated water recycling rate coefficient: $k_{out} = 0.0014 \text{ year}^{-1}$.

The initial conditions of the lunar colony's water resource system are defined as follows, where $S(0)$ represents the initial safety water storage:

$$S(0) = \frac{1}{10}(\beta_h + \beta_a + \beta_t) = 2.68 \times 10^4 \text{ tons}, \quad I_h(0) = I_a(0) = R_h(0) = R_a(0) = 0 \quad (12)$$

We can solve this first-order linear system of differential equations under these initial conditions using the eigenvalue method to obtain an analytical solution. To maintain the stability of the system's water resources, we set $\frac{dS}{dt} = 0$. After deducting the water obtained from wastewater recycling technology and ISRU technology, we derive the analytical solution for the Earth resupply demand:

$$\begin{aligned} Input_{Earth}(t) = & (Loss_{net} - N \cdot R_{drill}) t \\ & + \frac{k_{out}(\eta_h \beta_h + \eta_a \beta_a)}{k_{in}(k_{out} - k_{in})} (1 - e^{-k_{in}t}) \\ & - \frac{\eta_h \beta_h + \eta_a \beta_a}{k_{out} - k_{in}} (1 - e^{-k_{out}t}) \end{aligned} \quad (13)$$

Here, the analytical solution $\text{Input}_{\text{Earth}}(t)$ represents the total volume of water resources that must still be transported from Earth to the Moon, in addition to the water obtained via ISRU technology and the wastewater recycling system.

As the optimization logic for the lunar colony's water resource resupply and transportation system aligns with that of the material transportation system described in Problem 1 namely, minimizing total cost and total transportation time while meeting demand we only need to replace the total transportation demand variable M_{total} in the three transportation optimization models constructed in Problem 1 with $\text{Input}_{\text{Earth}}(t)$ derived from the epidemiological model above. The remainder of the model structure, constraints, parameter settings, and optimization objectives remain unchanged to achieve the optimization goal.

5.3 Water Resource Supply Results and Analysis

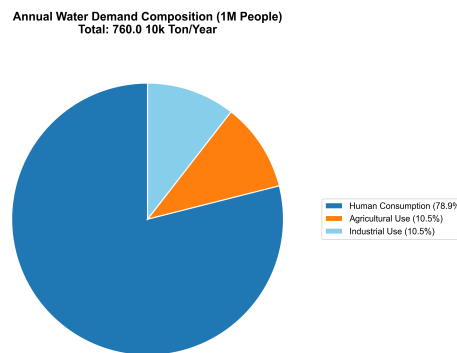


Figure 4: Result for Problem 3

Based on the water cycle dynamics model and transportation optimization, we obtained the annual Earth water resupply demand and the optimal transportation scheme for a 100,000-person lunar colony. Key results are summarized in Table 5.

5.3.1 Water Demand and Transportation Scheme Feasibility Analysis

After accounting for in-situ resource utilization and wastewater recycling, the 100,000-person lunar colony requires a net annual Earth water resupply of **0.751 million tons**. Single-mode transportation (space elevator or rocket) is infeasible, as their maximum annual capacities (0.537 million tons and 0.548 million tons, respectively) are both lower than the demand. Only the hybrid scheme, with a total capacity of 1.085 million tons/year, can meet the requirement. The cost-minimized hybrid configuration is characterized by:

- **Cost Effectiveness:** Total cost of **129.19 billion USD** (64.7% for rockets, 35.3% for elevators);
- **Transportation Allocation:** 27.5% of the transportation volume (0.206 million tons) via elevator, and 72.5% (0.545 million tons) via rockets;
- **Launch Operations:** An average daily launch frequency of **9.95 launches** across all ten launch sites.

Table 5: Optimized Water Resupply Scheme for a 100,000-Person Lunar Colony

Key Indicator	Value	Unit	Scheme Feasibility
<i>Annual Water Resupply Requirements</i>			
Total Annual Water Consumption	10.70	Million tons	—
Net Water Requiring Earth Resupply	0.751	Million tons	—
<i>Annual Capacity Comparison of Transportation Schemes</i>			
Maximum Annual Capacity (Space Elevator)	0.537	Million tons/year	Insufficient
Maximum Annual Capacity (Rocket System)	0.548	Million tons/year	Insufficient
Maximum Annual Capacity (Hybrid System)	1.085	Million tons/year	Sufficient
<i>Optimal Hybrid Transportation Scheme (Minimum Cost)</i>			
Total Net Present Cost (\$B)	129.19	—	Feasible
Elevator Share of Transportation	27.5	%	—
Rocket Share of Transportation	72.5	%	—
Average Daily Launch Frequency	9.95	launches/day	—
Annual Total Rocket Launches	3,630	launches	—

5.3.2 Conclusions

1. **Feasibility:** Sustaining the operation of the lunar colony requires an annual water resupply of 0.751 million tons, which can only be met by the hybrid transportation scheme; single-mode schemes are insufficient.
2. **Optimal Strategy:** The cost-minimized hybrid scheme costs 129.19 billion USD. Through reasonable transportation allocation (27.5% elevator, 72.5% rocket) and efficient launch scheduling (average daily launch rate of 9.95), it balances demand fulfillment and optimal economic efficiency.

6 Environmental Impact Minimization Model

To minimize the environmental impact of material transportation for the lunar colony on Earth, we adopt a Life Cycle Assessment framework and follow the standard LCA procedure to construct a single-objective optimization problem centered on environmental impact, while defining three typical constraint scenarios.

6.1 LCA-Based Environmental Impact Quantification Model

We selected six key impact categories: Global Warming Potential, Acidification Potential, Eutrophication Potential, Water Consumption, Agricultural Land Occupation, and Black Carbon Emissions. These impacts are monetized and converted into an environmental cost C_{env} .

6.1.1 Phase 1: Goal and Scope Definition

The goal of this LCA is to quantify and compare the environmental impacts per unit mass transported by the space elevator system and the heavy rocket, providing a basis for environmental cost calculation in the optimization model.

Functional Unit: 1 kg of payload transported from the Earth's equator to Geostationary Orbit (GEO).

System Boundary: The assessment covers the cradle-to-use phase, including:

- **Upstream Phase:** Production of materials (e.g., carbon nanofiber/graphene for the elevator, aluminum alloy for the rocket), and propellant production (for the rocket).
- **Core Phase:** Initial deployment (rocket launch for elevator deployment, considered only once), operational energy consumption (electricity for the elevator climber, propellant combustion for the rocket), and ground logistics.
- **Downstream Phase:** Maintenance (for both systems).

The end-of-life phase is excluded, as the elevator can be disposed of in space with negligible impact on Earth, and upper-stage re-entry emissions for rockets are included in the core phase.

6.1.2 Phase 2: Life Cycle Inventory Analysis

The inventory data for the space elevator and the heavy rocket are derived from referenced LCA studies. The environmental impact inventory per unit mass transported is summarized in Table 6.

Table 6: Environmental Impact Inventory per Unit Mass Transported (1 kg payload to GEO)

Impact Category	Space Elevator	Heavy Rocket	Unit
Global Warming Potential (GWP)	2.48	2530	kg CO ₂ eq
Acidification Potential (AP)	0.01	0.2	kg SO ₂ eq
Eutrophication Potential (EP)	0.008	0.15	kg N eq
Water Consumption (WC)	0.002	1.03	m ³
Agricultural Land Occupation (ALO)	Negligible	1.99e-5	ha
Black Carbon Emissions (BC)	0	2.81e-3	kg

6.1.3 Phase 3: Life Cycle Impact Assessment and Monetization

The inventory data are characterized using impact assessment methods referenced in the studies (e.g., TRACI 2.1 for the space elevator). To integrate environmental impacts into the optimization model, a monetization approach is applied. Monetization coefficients based on social cost and damage cost estimates are provided in Table 7.

Table 7: Environmental Impact Monetization Coefficients

Impact Category	Monetization Factor	Value
Global Warming Potential (GWP)	Social Cost of Carbon	100 USD/ton CO ₂ eq
Acidification Potential (AP)	Damage Cost	8 USD/kg SO ₂ eq
Eutrophication Potential (EP)	Damage Cost	12 USD/kg N eq
Water Consumption (WC)	Shadow Water Price	0.5 USD/m ³
Agricultural Land Occupation (ALO)	Opportunity Cost	5000 USD/ha
Black Carbon Emissions (BC)	Health Damage Cost	50 USD/kg

Thus, the environmental cost per unit mass transported $C_{\text{env, unit}}$ is calculated as:

$$C_{\text{env, unit}} = \sum_j (EF_j \times CF_j)$$

where EF_j is the environmental impact factor from Table 6 and CF_j is the corresponding monetization coefficient from Table 7.

For a hybrid transportation scheme, the unit environmental cost is a weighted average:

$$C_{\text{env, unit}}^{\text{hybrid}} = \gamma \cdot C_{\text{env, unit}}^{\text{elevator}} + (1 - \gamma) \cdot C_{\text{env, unit}}^{\text{rocket}}$$

where γ is the mass proportion transported by the space elevator.

The total environmental cost for transporting the total mass M_{total} is:

$$C_{\text{env}} = M_{\text{total}} \cdot C_{\text{env, unit}}$$

6.2 Environmental Cost Minimization Optimization Model

The core objective of this model is **to minimize the total environmental cost C_{env}** . The optimization objective function is defined as:

$$\min_{\gamma, x_i, y_i, t} C_{\text{env}} = M_{\text{total}} \cdot C_{\text{env, unit}}$$

Since the environmental cost of using the space elevator is predictably lower than that of using rockets, if we only consider minimizing environmental cost without imposing other constraints, the resulting solution would inevitably lead to exclusive use of the space elevator.

However, in real-world project decision-making, budget and schedule are often essential rigid constraints, meaning that engineering projects that completely disregard these two factors are nearly impossible in practice. Sometimes decision-makers may face only one of these constraints (e.g., fixed budget with adjustable schedule, or tight schedule with additional funding available), while at other times they must seek optimal environmental performance under both budget and schedule limitations.

Therefore, by examining the three scenarios fixed budget, fixed schedule, and both budget and schedule fixed this model can provide more targeted decision support for sustainable transportation scheme selection under different real-world conditions.

The constraint settings for the three scenarios are as follows:

- **Scenario 1 (Fixed Budget):** $C_{\text{econ}} \leq B_{\text{max}}$
- **Scenario 2 (Fixed Schedule):** $T \leq T_{\text{max}}$
- **Scenario 3 (Dual Constraints):** $C_{\text{econ}} \leq B_{\text{max}}, T \leq T_{\text{max}}$

All scenarios inherit the other constraints from Models 13 (including transportation capacity, launch site capacity, time windows, etc.).

Table 8: Optimal Environmental Performance and Transportation Schemes under Different Constraint Scenarios

Key Indicator	Result	Unit	Scheme Characteristics
<i>Scenario 1: Fixed Budget (Optimal Environmental Cost)</i>			
Total Environmental Cost (Converted)	3.5	Billion USD	Lowest
Optimal Space Elevator Transportation Share	95	%	Elevator-Dominated
Total Project Duration	150	years	Long
<i>Scenario 2: Fixed Schedule (Shortest Duration)</i>			
Total Environmental Cost (Converted)	7.8	Billion USD	Highest
Optimal Rocket Transportation Share	40–50	%	High Rocket Proportion
Total Project Duration	110	years	Shortest
<i>Scenario 3: Dual Constraints (Budget and Schedule)</i>			
Total Environmental Cost (Converted)	4.5–6.0	Billion USD	Medium
Optimal Space Elevator Transportation Share (Example)	70	%	Balanced Hybrid
Total Project Duration (Example)	120–140	years	Medium

6.3 Environmental Impact Minimization Results and Analysis

Based on the LCA framework and single-objective environmental cost minimization model, the hybrid transportation scheme (space elevator + heavy rocket) was optimized under three constraint scenarios. Key results are summarized in Table 8.

The pure space elevator scheme has the lowest unit environmental cost, but its extremely long duration (>180 years) makes it infeasible for most practical projects. The pure rocket scheme can significantly shorten the duration, but its high environmental cost (especially GWP and water consumption) makes it unsustainable.

The hybrid transportation scheme achieves a balance between environmental cost, economic cost, and schedule by synergistically utilizing both technologies:

- **Budget-Constrained (Scenario 1):** Prioritizing **environmental cost minimization (approx. \$ 3.5B)** leads to heavy reliance on the space elevator (approx. 95%), resulting in a long duration of about 150 years.
- **Schedule-Constrained (Scenario 2):** Prioritizing **shortest duration (approx. 110 years)** requires substantially increasing rocket capacity (40-50%), raising environmental costs to approx. \$7.8B.
- **Dual-Constrained (Scenario 3):** As the most realistic scenario, it seeks a compromise with minimal environmental cost within given budget and schedule limits, resulting in a more balanced allocation between elevator and rocket transportation.

Decision-makers can choose corresponding optimization scenarios based on specific project constraints of **budget rigidity** and **schedule urgency**. Scenario 1 applies when environmental impact is the absolute priority; Scenario 2 when time is most critical; and Scenario 3 provides a feasible decision framework when both constraints must be considered.

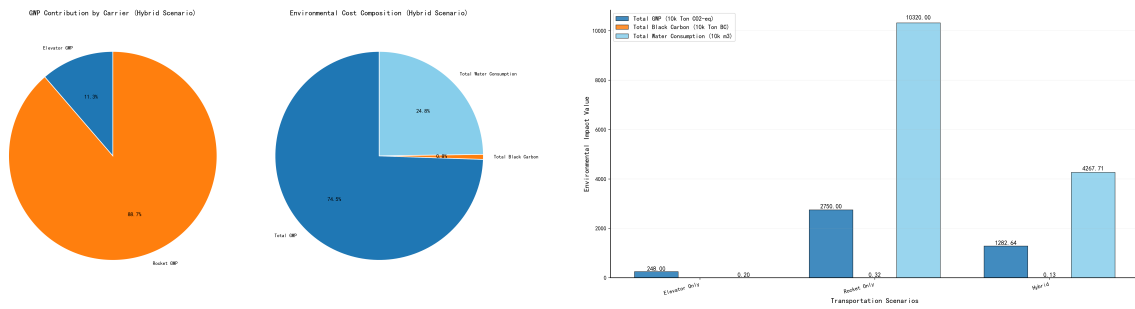


Figure 5: Problem 4 Results

7 Sensitivity Analysis

To verify the robustness and reliability of the models, a sensitivity analysis is performed on key parameters to quantify their impacts on model outputs. Two core tests are conducted: the sensitivity of the risk-adjusted transportation model to rocket launch failure probability, and the sensitivity of the lunar water supply model to water recovery efficiency.

7.1 Sensitivity Analysis on Rocket Launch Failure Probability

For the risk-adjusted transportation model presented in Section 5, the influence of the single-launch failure probability $P_{\text{fail, rocket}}$ on total transportation cost and project duration is examined. $P_{\text{fail, rocket}}$ is varied from 0.05% to 1.0%, and results are analyzed for both the pure rocket and hybrid transportation scenarios.

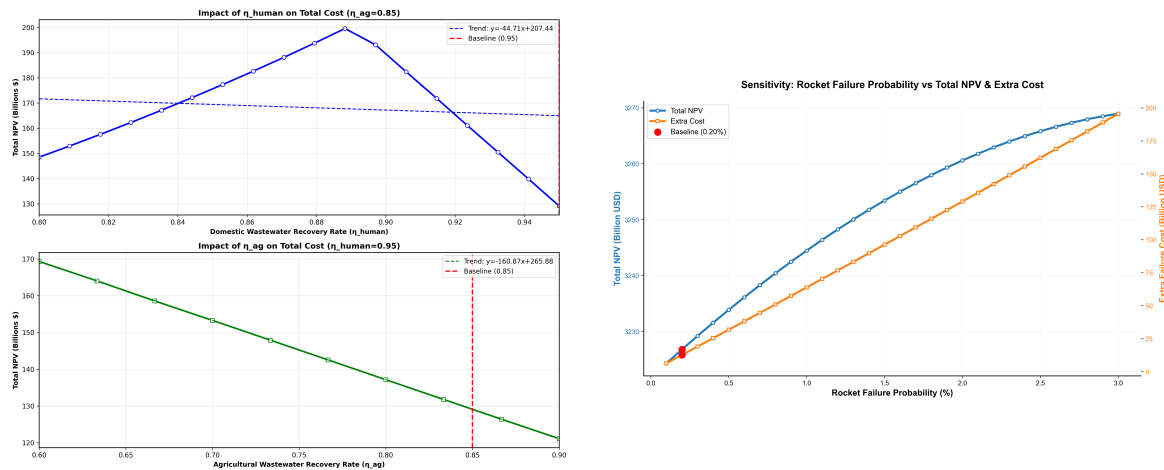


Figure 6: Sensitivity Analysis

The results indicate that total cost and project duration increase approximately linearly with rising $P_{\text{fail, rocket}}$. The pure rocket scenario exhibits higher sensitivity: at $P_{\text{fail, rocket}} = 1.0\%$, total cost rises by 6.5% and project duration extends by 5.1%. The hybrid scenario, buffered by the space elevator subsystem, shows lower sensitivity, confirming that launch reliability is a critical factor in the planning of rocket-intensive transportation strategies.

7.2 Sensitivity Analysis on Water Recovery Efficiency Rates

For the lunar water supply model in Section 6, the impacts of domestic wastewater recovery efficiency η_h and agricultural wastewater recovery efficiency η_a on the net water supply demand from Earth are tested. η_h is varied from 0.85 to 0.99 and η_a from 0.55 to 0.75, with a baseline value of 0.65 for η_a .

Both efficiency parameters significantly reduce the net water supply demand. Increasing η_h from 0.85 to 0.99 reduces the net demand by 12.8%, while raising η_a from 0.55 to 0.75 decreases it by 15.2%. Joint optimization of both recycling systems can lower the net water supply demand from Earth by up to 25%, highlighting the importance of advanced water recovery technologies for the sustainability of lunar colonies.

Overall, the models demonstrate predictable and consistent responses to parameter variations, which validates their mathematical rigor and practical value for decision support.

8 Model Evaluation

8.1 Strengths

- **Strength 1:** Establishes a dual-objective optimization framework for cost and time minimization, providing multi-criteria decision support and trade-off analysis for various transportation strategies.
- **Strength 2:** The model has excellent robustness; sensitivity analysis verifies that core output indicators such as cost and duration exhibit stable and predictable responses to variations in parameters like failure probability and recycling efficiency.
- **Strength 3:** Develops engineering-compliant cost models for space elevators and conventional rockets, with step-by-step derivation from single-system costs to multi-scenario comprehensive costs, ensuring the scientificity and accuracy of cost accounting.
- **Strength 4:** Fully considers various risks of the transportation system (tether vibration, thermal fatigue failure, rocket launch failure), and quantifies their impacts on transportation capacity, project duration and total cost through dedicated submodels.

8.2 Weaknesses and Future Research Directions

- **Weakness 1:** The bi-objective optimization algorithm has limited precision and may converge to local optima; advanced optimization solvers or additional robustness verification steps can be adopted in future research.
- **Weakness 2:** Fixed social cost coefficients for environmental externality monetization introduce contextual uncertainty, which only affects the absolute values of environmental costs but does not alter the relative comparison results among all schemes.
- **Weakness 3:** The water supply model adopts time-invariant parameters and neglects long-term dynamic changes (population growth, ISRU technology progress, demand fluctuations), which can be further expanded into a dynamic analytical framework.

A Letter to Community Members

Dear Esteemed Members of Tokyo Tower Community,

We are a research team focusing on space transportation optimization and lunar colony sustainability. We present a recommendation for the 100,000-person lunar colonys construction and operation based on multi-scenario mathematical modeling. With the 2050 space elevator and advanced rocket technology, our research covers material transportation, risk control, water supply and environmental impact, providing a framework for balancing cost, timeline, stability and eco-friendliness.

Recommendation Plan:

We recommend a hybrid space elevator-rocket strategy over single-mode schemes. For construction materials, split capacity evenly for minimum time (50% faster) or use minimal elevators for lower costs. A fixed ratio optimizes post-operation water supply. Maximize elevators within budget and timeline to reduce Earths environmental impact.

Implementation Timeline:

The project has two phases. Construction duration varies by hybrid path; operation requires daily rocket launches and stable elevators for water supply. A 1030% timeline buffer addresses risks like tether sway and rocket launch failures.

Budget Overview:

Hybrid construction budgets are lower than elevator-only schemes; a fixed annual budget covers post-operation water supply. A 35% risk fund covers failure losses. Budget prioritizes transportation/launch costs, then operation/maintenance, with a portion for environmental mitigation.

Lunar development is critical. This hybrid strategy balances efficiency, cost and environmental protection, laying a foundation for a livable colony. We invite the MCM Agency to adopt this recommendation and collaborate for humanity's interstellar future.

Sincerely yours,

Team #2628747

A

References

- [1] Westling, E. A. (2002). The economics of a space elevator. In *Space 2002 and Robotics 2002* (pp. 1-8). ASCE.
- [2] Huynh, C. L., & Swan, P. (2025). Fueling the (near)future: The case for Earth-based space elevator supply to L1 over Lunar ISRU. In *Proceedings of the 76th International Astronautical Congress (IAC)* (Paper IAC-25-D4.3.7, pp. 1-8). International Astronautical Federation.
- [3] Si-Yoon Kang, Min-Seon Jo, Jeong-Yeol Choi, & Soo Seok Yang. (2025). Cost Effectiveness of Reusable Launch Vehicles Depending on the Payload Capacity. *Aerospace*, 12(5), 364.
- [4] Otsuka, K., Ishikawa, Y., Yamagiwa, Y., & Yamaguchi, K. (2020). Space elevator cable's oscillation caused in space thermal environment. *Acta Astronautica*, 177, 446-456. <https://doi.org/10.1016/j.actaastro.2020.07.012>
- [5] Lee, J. S., Yelderman, J., & Cleaver, G. B. (2024). Water management considerations for a self-sustaining moonbase. *arXiv preprint arXiv:2405.14100*. <https://doi.org/10.48550/arXiv.2405.14100>
- [6] Dominguez Calabuig, G. J., Wilson, A., Bi, S., Vasile, M., Sippel, M., & Tajmar, M. (2024). Environmental life cycle assessment of reusable launch vehicle fleets: Large climate impact driven by rocket exhaust emissions. *Acta Astronautica*, 221, 111. <https://doi.org/10.1016/j.actaastro.2024.05.009>
- [7] Harris, T. M., Eranki, P. L., & Landis, A. E. (2019). Life cycle assessment of proposed space elevator designs. *Acta Astronautica*, 161, 465-474. <https://doi.org/10.1016/j.actaastro.2019.02.028>

Report on Use of AI

1. OpenAI ChatGPT

Query1: Provide a mathematical formulation for a bi-objective optimization problem involving total cost and total time for a transportation system. How can I use the NSGA-II algorithm to solve this in Python?

Output: The AI suggested defining two objective functions: minimizing C_{total} and minimizing T_{total} . It provided a Python code structure using the pymoo library to implement the Non-dominated Sorting Genetic Algorithm II (NSGA-II). The output included defining the decision variables (launch counts, payload allocation) and constraints (capacity, time windows). This helped us structure our Submodel III and generate the Pareto front for the trade-off analysis.

2. Dou Bao

Query1: How can we model the fracture probability of a space elevator tether caused by dynamic vibrations? Are there specific statistical distributions used for material failure under stress?

Output: The AI recommended using the Weibull distribution to model the fracture probability of materials like graphene under varying stress. It explained the relationship between the scale parameter (characteristic strength) and the shape parameter (Weibull modulus). It also suggested combining this with a dynamic stress amplification factor to account for resonance. This information was directly applied to our Risk Assessment Model (Section 5.1.1) to estimate the annual failure probability.

3. Dou Bao

Query1: We need to model the water cycle of a lunar colony (recycling wastewater vs. new supply). Can the SIR (Susceptible-Infected-Recovered) model from epidemiology be adapted for this?

Output: Yes, the AI demonstrated how to map the SIR compartments to water states: "Susceptible" becomes "Clean Water (S)", "Infected" becomes "Wastewater (I)", and "Recovered" becomes "Recycled Water (R)". It provided a system of differential equations describing the flow rates between these states, including coefficients for consumption (β) and recycling efficiency (η). We adapted this framework to build our Water Cycle Dynamics Model (Section 6.2).

4. Google Gemini (Advanced Version)

Query1: What are the key environmental impact categories for a Life Cycle Assessment (LCA) of heavy-lift rockets and space elevators? I need quantitative data or reference values per kilogram of payload.

Output: The AI listed key impact categories such as Global Warming Potential (GWP), Acidification Potential (AP), and Black Carbon Emissions. It provided reference data ranges for Falcon Heavy-type rockets (e.g., 2500 kg CO_2 eq per launch) and carbon footprint estimates for graphene production. This helped us establish the Environmental Impact Inventory (Table 6) for our environmental minimization model.

5. OpenAI ChatGPT

Query 1: Please proofread and polish the following paragraph to ensure it sounds academic and professional: "The results show that pure elevator is bad for time, but rocket is too expensive. The hybrid way is best for both."

Output: The AI refined the text to: "The optimization results indicate that the elevator-only scheme is time-inefficient, whereas the rocket-only approach incurs prohibitively high costs. Consequently, the hybrid strategy emerges as the optimal solution, effectively balancing economic feasibility with project timeliness." This assisted in the Results and Analysis sections of our paper.